Gibbs Random Field-Based Vector Quantization

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Vector Quantization

- Mapping Q from R^n into a finite subset Y of R^n:

$$Q : R^n \rightarrow Y$$

- $Y = (\boldsymbol{a}_k : k = 1,\dots, K)$, the set of prototypes.
- Applications: speech coding, image compression, **image segmentation**

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Motivation

Reformulate classical VQ used for **image segmentation** to incorporate contextual constraints and a stochastic iterative algorithm

	Type of Annealer and Labeling	
	Deterministic	**Stochastic**
No Contextual Constraints	A. Standard k-means	B. Probabilistic k-means
Contextual Constraints	C. K-means with local constraints	D. Probabilistic k-means with local constraints

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A Standard VQ Algorithm: K-Means

$X=(\boldsymbol{x}_{i,j} : i = 1,\dots, M, j = 1,\dots, N)$, $M\times N$.
$L =(l_{i,j} : i = 1,\dots, M, j = 1,\dots, N)$, labels.

- Choose the number of clusters K.
- Set initial cluster prototypes $Y = (\boldsymbol{a}_k : k = 1,\dots, K)$.
- Compute $d(\boldsymbol{x}_{i,j}, \boldsymbol{a}_k)$ for all i, j and k.
- Assign $l_{i,j} = k$ if $d(\boldsymbol{x}_{i,j}, \boldsymbol{a}_k) < d(\boldsymbol{x}_{i,j}, \boldsymbol{a}_{k'})$ for all $k\neq k'$.
- Update prototypes using $\boldsymbol{a}^*_k = 1/N_k\cdot(\Sigma_{i,j}\, \boldsymbol{x}_{i,j}\cdot(l_{i,j} =k))$
- If clusters consistent stop, else go to step 3.

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Gibbs Random Fields

- Gibbs/Markov random fields provide a natural way of modeling context dependencies between, e.g., image pixels of correlated local features.
- Key design issues:
  - Objective function (segmentation models)
  - Algorithm to find the "optimal" solution. In this case, the Gibbs Sampler and Simulated Annealing are used.

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Objective Function Without Context → Algorithms A and B

$$E[\{l_{i,j}, \mathbf{a}_k\}] = \sum_{i,j} d(\mathbf{a}_{l_{i,j}}, \mathbf{x}_{i,j})$$

where $d(\cdot,\cdot)$ is a similarity measure (typically the Euclidean distance), where pixel (i,j) is assigned a label $0 \leq l_{i,j} \leq N$ corresponding to a cluster/region prototype $\boldsymbol{a}_{l_{i,j}}$.

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Objective Function With Context → Algorithms C and D

$$E[\{l_{i,j}, \mathbf{a}\}] = \sum_{i,j} \{\gamma\, d(\mathbf{a}_{l_{i,j}}, \mathbf{x}_{i,j}) + \eta\,[(1-\delta_{l_{i,j}, l_{i+1,j}}) + (1-\delta_{l_{i,j}, l_{i,j+1}})]\}$$

where $\delta_{l_{i,j}\, l_{i+1,j}}$ indicates whether pixels (i, j) and $(i+1, j)$ have the same label, γ and η control region homogeneity and fragmentation

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Algorithms A and C vs. B and D

- Algorithms A and C are deterministic:
  - Label of closest prototype assigned to pixel
  - Region prototype is the vector "mean" of the pixels with the given label
- Algorithms B and D are stochastic, use Gibbs Sampling:
  - Label of closest likely prototype assigned to pixel
  - Most likely pixel from labeled region to be cluster prototype assigned as new cluster prototype

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Gibbs Sampler

- Sampling labels:

  $l^*_{i,j}$ ← sample from multinomial[$\alpha_1,\ldots,\alpha_K$]

$$\alpha_k \propto e^{[d(\mathbf{a}_{l_{i,j}},\mathbf{x}_{i,j})+U(\text{neighbors})]/T}$$

- Sampling region prototypes:

  a^*_k ← sample from $\{x_{i,j} : l_{i,j} = k\}$

$$p(\mathbf{x}_{i,j}) \propto e^{-d(\mathbf{a}_k,\mathbf{x}_{i,j})/T}$$

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Experiments

Perform color image segmentation using vector angle instead of Euclidean distance

	Type of Annealer and Labeling	
	Deterministic	**Stochastic**
No Contextual Constraints	Mixture of Principal Components	Adaptive Gibbs Random Field
Contextual Constraints	MPC with local constraints	Adaptive Gibbs Random Field with local constraints

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Results: Color Model

- Highlight invariant transformation:

$$\mathbf{c}'_{i,j} = \begin{bmatrix} R'_{i,j} \\ G'_{i,j} \\ B'_{i,j} \end{bmatrix} = \begin{bmatrix} R_{i,j} \\ G_{i,j} \\ B_{i,j} \end{bmatrix} - \frac{1}{3}\left(R_{i,j} + G_{i,j} + B_{i,j}\right)$$

- Shading invariant transformation:

$$d_\Theta(\mathbf{c}'_{i,j},\mathbf{a}'_{l_{i,j}}) = \frac{\left\langle \mathbf{c}'_{i,j},\mathbf{a}'_{l_{i,j}} \right\rangle}{\left\|\mathbf{c}'_{i,j}\right\| \cdot \left\|\mathbf{a}'_{l_{i,j}}\right\|}$$

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Mixture of Principal Components

	Algorithm	
	K-Means	**Mixture of Principal Components**
Similarity Measure	Euclidean distance	Vector angle
Prototype Calculation	Vector mean	First principal component of cluster data covariance matrix

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Results: Vector Angle Reliability

- For pixels with small values, the inner vector product can vary considerably with small variations in pixel value
- The reliability of the vector angle similarity measure is computed based on Monte Carlo means:

$$u_{i,j} = \min\left\{ \frac{1}{\text{var}(d_\Theta(\mathbf{c}'_{i,j},\mathbf{a}'_{l_{i,j}}))}, \frac{1}{\text{var}_N(d_\Theta(\mathbf{c}'_{i,j},\mathbf{a}'_{l_{i,j}}))} \right\}$$

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Modified Model for C and D

$$E[\{l_{i,j},\mathbf{a}\}] = \sum_{i,j}\{\gamma u_{i,j} d(\mathbf{a}_{l_{i,j}},\mathbf{x}_{i,j}) + \eta[(1-\delta_{l_{i,j},l_{i+1,j}}) + (1-\delta_{l_{i,j},l_{i,j+1}})]\}$$

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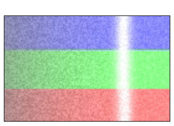
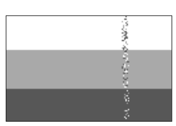
Results

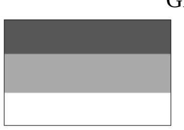


Original

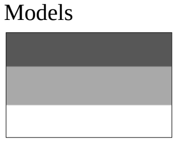


MPC Result

Global GRF Models



Adaptive GRF



Adaptive GRF (weighed)

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Conclusions

- A Gibbs Random Field can be designed to effectively model VQ-based image segmentation with global parameters (i.e. $\boldsymbol{a}_{l_{i,j}}$).
- Both global and local constraints provide a robust framework for image segmentation.
- The disadvantage is computational cost.