FOVEATED MULTISCALE MODELS FOR LARGE-SCALE ESTIMATION

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ABSTRACT

Efficient, large-scale estimation methods such as nested dissection or multiscale estimation rely on a divide-and-conquer strategy, in which a statistical problem is conditionally broken into smaller pieces. This conditional decorrelation is not possible for arbitrarily large problems due to issues of computational complexity and numerical stability. Given the growing interest in global-scale remote sensing problems (or even three-dimensional problems), in this summary we develop a class of estimators with more promising asymptotic computational properties.

1. INTRODUCTION

Heightened environmental awareness and concerns have led to an explosion in the quantity of remotely-sensed data, leading to more ever-larger problems requiring statistical estimation (for example, to remove irregularities in sampling or to detect anomalous behaviour).

Most efficient, large-scale estimation methods (e.g., nested dissection[5, 6] or multiscale estimation[1]) rely on some sort of divide-and-conquer strategy: a state vector is found which conditionally breaks the problem into smaller pieces. As the size of the underlying problem grows (Fig. 1), this first conditional division becomes increasingly problematic:

- The overall computationally complexity grows as the cube of the length of this first state.
- More significantly, the covariance associated with the state vector becomes more poorly conditioned as the state length grows, such that solutions become numerically unstable beyond a certain size.

Given the current interest in global-scale problems, the above asymptotic weaknesses motivate alternative methods for large-scale estimation; this paper presents one such alternative.

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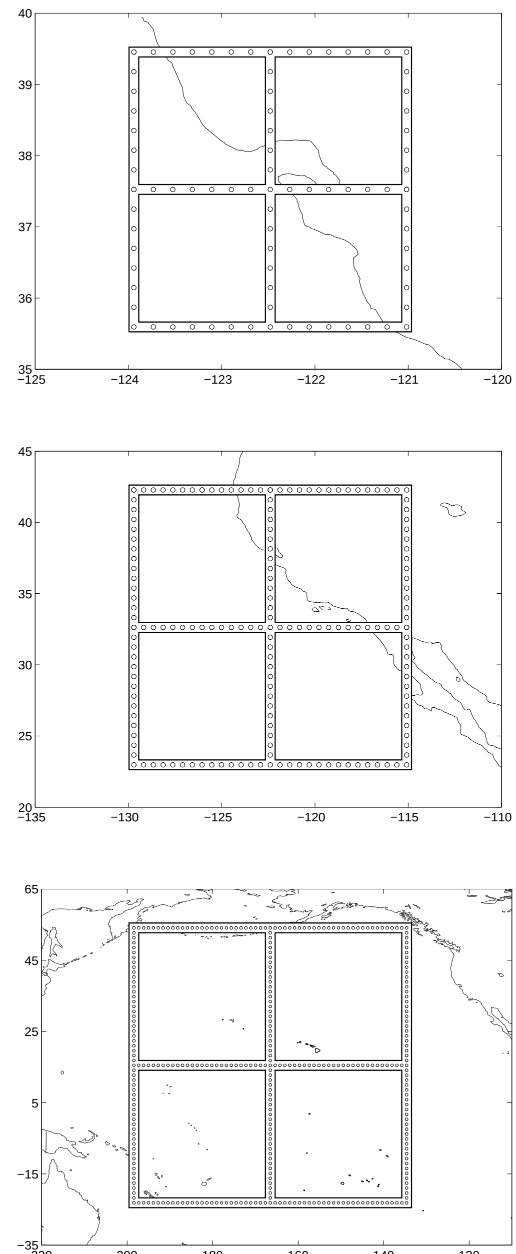


Fig. 1. For how large a domain is divide-and-conquer computationally and numerically feasible ...?

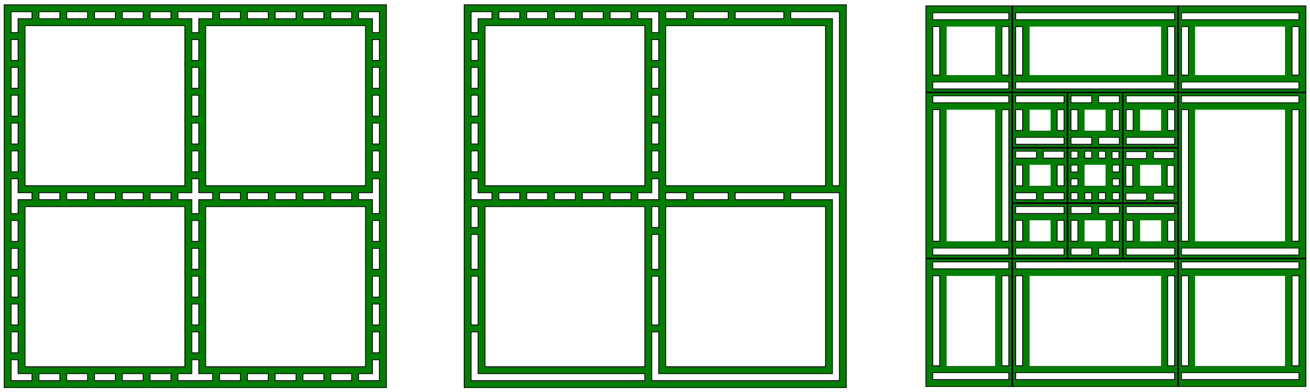


Fig. 2. How many separate models are desired for a given region? A single model (left), one model per quadrant (middle), or a large number of "foveated" models, each capturing only a small subset (right).

2. MULTIPLE TREES

As has been discussed in an earlier paper[4], it may be possible to solve the estimation problem using $m > 1$ models (Fig. 2), where each model is responsible for a subset of the overall region; the desired estimates result from the collaging of the estimates of the individual models. The fundamental motivation behind this idea is as follows: a model designed to estimate (say) quadrant 1 does not need to conditionally decorrelate all four quadrants, rather only those statistical aspects of quadrants 2,3,4 relevant to estimating quadrant 1 need to be kept. Since computational effort scales as $\mathcal{O}(n^3)$, only a 37% reduction in state dimension is required to compensate for the $m = 4$ increase in the number of models. In addition to the computational reduction, the shortened state dimension typically leads to reduced numerical instabilities.

Arguably a *local* estimation scheme, in which estimates are based on measurements within some local vicinity, could yield similar computational and numerical benefits, however continuing to use multiscale models and estimation leads to substantial benefits:

- Most significantly, each estimate is based on *all* of the measurements; that is, the approach is *not* local. Furthermore, the global nature of the models permits data fusion with non-local measurements.
- Other advantages of the multiscale model are maintained: efficient likelihood estimation, a stochastic realization theory, and a base of existing models and applications.

With the above framework in place, three issues need to be settled for a preliminary implementation:

(a) How many models m is desirable (or "optimal")?

(b) What is the nature of the model (i.e., what statistical information is kept)?

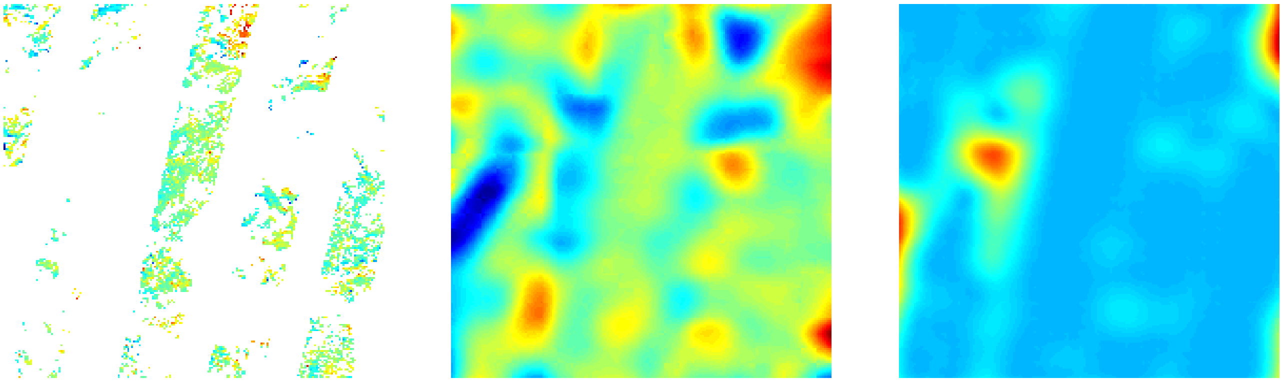
(c) How is the model placed into the multiscale framework?

None of the above is made clear by Fig. 2(right), which presents (at best) highly qualitative answers.

The answer to (a) follows from (b), since the dependence of computational complexity on m is easily computed once the model details are known.

We have chosen (tentatively) a stratified or foveated model, in which the $p \times p$ square region to be estimated is surrounded by several (here three) concentric regions, modeled with progressively coarser statistical fidelity. The "thickness" of the inner two concentric regions is set approximately to the correlation length of the underlying statistics. As the problem grows asymptotically, only m changes, *not* the size p of the detailed region, as opposed to the traditional multiscale approach, in which $m = 1$ and p grows with the problem size. It is this distinction which accounts for the promising computational properties of our revised approach.

Details of the multiscale implementation go beyond the scope of this summary. We depart significantly from the standard quadtree model — the root node is based on the $p \times p$ region of interest, with nine descendants (instead of four) at three scales to construct the concentric sets in the model. The detailed workings of multiscale estimation algorithm are, however, unchanged from before[1, 2, 7].



Infrared Measurements | Temperature Estimates | Estimation Error Variances

Fig. 3. Estimation results (200×200 pixels, isotropic Gaussian prior), given pointwise measurements of ocean surface temperature in the equatorial Pacific. The measurements and estimates are mean-removed, and vary over about ±1.5Kelvin.

3. RESULTS

Fig. 3 shows one example of applying the multiple-tree approach to a remote-sensing problem of current interest[3] in climate-modeling and climate-change studies — estimating the surface temperature of the ocean from infrared measurements (left), which are sparse and are taken in bands. The estimates (middle) are based on 64 models within an overlapped multiscale framework[7]. Overall, 200×200 estimates and error statistics were computed.

A number of interesting challenges remain, which shall be reported upon in the final paper and in the conference presentation:

- Realistic benchmarks: Our current experimental implementation runs each of the models separately, whereas in fact the m estimation steps share a great deal in common. This commonality needs to be studied and exploited.

- In order to produce estimates of a group of pixels, what statistical information really needs to be preserved from the rest of the domain? The answer will clearly be prior-model dependent, but would be worth exploring for some models (e.g., anisotropic-Gaussian) in widespread use in remote sensing.

- When computing estimates far away from any measurements (i.e., extrapolating), the values of the estimates, although possibly statistically insignificant, may be highly sensitive to the choice of prior model; consequently, the extrapolated values computed by neighbouring models may vary significantly. It is not clear how to properly mosaic in the face of such variations.

4. REFERENCES

[1] P. Fieguth, W. Karl, A. Willsky, C. Wunsch, "Multiresolution Optimal Interpolation and Statistical Analysis of TOPEX/POSEIDON Satellite Altimetry", *IEEE GRS* (33) #2, pp.280–292, 1995

[2] P. Fieguth, D. Menemenlis, C. Wunsch, A. Willsky, "Adaptation of a Fast Optimal Interpolation Algorithm to the Mapping of Oceanographic Data", *JGR* (102) #C5, pp.10573–10584, 1997

[3] P. Fieguth, M. Allen, M. Murray, "Hierarchical Methods for Global-Scale Estimation Problems," *Canadian Conf. on Elec. & Comp. Eng.*, Waterloo, Canada, 1998

[4] P. Fieguth, "Multipole-Motivated Reduced-State Estimation," *IEEE ICIP'98*, Chicago, pp.MP5.08-1–4, 1998

[5] A. George, *Computer solution of large sparse positive definite systems,* Prentice-Hall, 1981.

[6] G. Golub, R. Plemmons, "Large-Scale Geodetic Least-Squares Adjustment by Dissection and Orthogonal Decomposition," in *Large Scale Matrix Problems* (Björk, Plemmons, Schneider eds.), North Holland, New York, pp.3–28, 1981

[7] W. Irving, P. Fieguth, A. Willsky, "An Overlapping Tree Approach to Multiscale Stochastic Modeling and Estimation", *IEEE Trans. on Image Processing* (6) #11, pp.1517–1529, 1997

[8] W. Irving, W. Karl, A. Willsky, "A Theory for Multiscale Stochastic Realization", *33rd Conference on Decision and Control*, Orlando, FL, pp.655–662, 1994