Lab 3

Due Nov 17, In Class

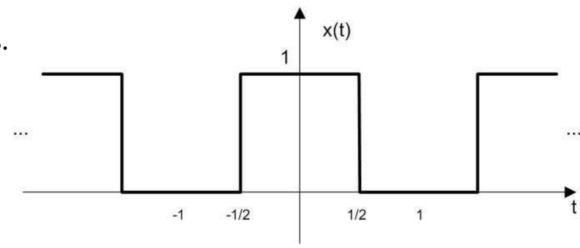
Question 1 takes a first look at decomposing a periodic signal into its Fourier Series representation. In Questions 2 and 3 we are interested in studying some basic properties of Fourier Transforms. You will need the following MATLAB script:

lab3_ft.m, which implements the continuous-time Fourier transform

1. **Fourier Series:** representation of periodic signals.

   In class we derived the Fourier series for a periodic square wave, here with a period $T = 2$.

   Write down w_0 and a_n.



   If we define the MATLAB variable `t = [-2:0.01:2]` then it is easy to plot the reconstructed signal with the first N terms in the series:

$$x(t) \approx \sum_{k=-N}^{N} a_k e^{jkw_0 t} = a_0 + 2\sum_{k=1}^{N} a_k cos(kw_0 t) \quad (1)$$

   Submit four plots (but no Matlab code), corresponding to the reconstructed square wave when $N = 1, 3, 10, 50$, using code something like

```
x = a(1)*cos(0*t); for k=1:N, x = x + 2*a(k+1)*cos(k*wo*t); end;
```

   (where, because MATLAB indexes vectors from 1 and not 0, we have a_0 = `a(1)`, a_n = `a(n+1)`)

2. **Consider the following two signals:**

$$x_1(t) = e^{-at}u(t) \qquad a > 0$$
$$x_2(t) = u(t+T) - u(t-T) \qquad T > 0$$

   Determine analytically the Fourier transform of these signals. Explain, briefly, the relationship between the time and frequency-domain signals as a and T are varied.

   Next, use *lab3_ft.m* to compute the Fourier transforms of these signals for $a = 1$ and $T = 1$. Produce a magnitude and phase plot for both $X_1(j\omega)$ and $X_2(j\omega)$. How do these results compare with the analytical solutions?

3. **The Importance of Phase**

   (a) We know that we can recover any discrete-time signal perfectly from its frequency representation, thus `ifft(fft(s))` will equal `s`, except for rounding errors.
   Create a delta signal `d = [1 zeros(1,127)];` then

   ```
   stem(real(ifft(fft(d))))
   ```

   clearly reproduces `d` (try it). But now randomize the frequency-domain phase (setting each phase to an independent, uniform random value between 0 and 2π, using `rand`), and look at the time-domain result. Is this a delta function, a shifted delta, or something very different? Do the randomization three times and plot each result.

   (b) The delta function is a bit of a worst-case. Repeat part a), but now with a triangular function

   ```
   t = [1:20 19:-1:1 zeros(1,100)];
   ```

   Find the time-domain signal corresponding to randomizing the phase in the frequency domain. Do this three times and plot each result.

   (c) For what sort of signal is the sensitivity to phase randomization the least? That is, speculate on what sort of time-domain signal distorts relatively little with changes in phase.