Lab 4

Due Dec. 1, In Class

1. **Signal Modulation**

   Consider a signal $x(t)$ which we wish to modulate (multiply) with two others $p_1(t), p_2(t)$:

   $$x(t) = \frac{\sin(\pi t)}{\pi t} \qquad p_1(t) = \cos(3\pi t) \qquad p_2(t) = u(t+1) - u(t-1)$$

   We are interested in the spectrum of signals $x(t) \cdot p_1(t)$, $x(t) \cdot p_2(t)$:

   (a) Based on our understanding of signal modulation and truncation, sketch the magnitude of the Fourier transform of the two modulated signals.

   (b) Create the two modulated signals in MATLAB and use *lab3_ft.m* to generate the spectrum. Plot the magnitude (absolute value) of the two spectra.

   How do the numerical and sketched results compare?

2. **Signal Processing in the Frequency Domain**

   You're given two different signals over time range `t = 0:0.01:20;`

   (a) A sum of two sine waves: `s = cos(t) + cos(2.5*t);`

   (b) A wideband signal: `w = conv(randn(size(t)),[1:80 79:-1:1]);`

   For each of the above signals examine the effect of:

   - Applying an ideal low-pass filter to the signal.
   - Applying an ideal high-pass filter to the signal.

   In MATLAB use the `fft` and `ifft` commands to move between the frequency domain (DTFS) and time domain. Accomplish the filtering by setting high or low frequency elements to zero (low-pass and high-pass filtering, respectively). For each signal, choose an appropriate intermediate cut-off frequency (the boundary between those frequency elements set to zero, and those left unchanged).

   Submit well labelled plots of the input signal in the time domain, the input spectrum in the frequency domain, and the low-pass and high-pass output signals in the time domain. Comment briefly on the results. For signal `s`, do the filtered signals look like perfect sinusoids?

3. **Comparing the Discrete-Time and Continous-Time Frequency Domains**

   For our comparison we'll use the familiar rectangular pulse $s(t) = u(t + T/2) - u(t - T/2)$

   (a) What is the spectrum of the CTFT $S(j\omega)$? Provide a rough sketch.

   (b) There is ambiguity in how to define $s(n)$ from $s(t)$: sampling rate, length of signal etc.
   We'll let $s(n)$ be a sequence of ones, followed by zeros: `s = [ones(1,`n_1`) zeros(1,`n_0`)];`
   The magnitude of the DTFS is then found as `S_mag = abs(fft(s));`

   Do nine tests, trying each of the nine combinations of $n_0 = 5, 50, 500$ and $n_1 = 5, 25, 55$. State qualitative interpretations of the resulting spectra. Plot the magnitude spectrum for $n_0 = 500, n_1 = 25$. Ensure that your horizontal axis is in units of ω (0 to 2π).

   We know that n_0 is the degree of zero-padding, affecting the density of frequency domain samples, whereas n_1 is the width of the rectangular pulse. Thus changing n_0 shouldn't change the spectrum at all, just how closely the samples are spaced. Show that this is true by superimposing the spectral magnitudes for

   n_0 `= 3,` n_1 = 15, plotted using discrete symbols, such as '*'
   n_0 `= 3000,` n_1 = 15, plotted as a line.