SD 252 Signals & Systems 2004

Midterm Examination

Professor Paul Fieguth

Time:	90 Minutes Schedule your time carefully.
Grading:	The exam is out of 100. The grade breakdown is shown in the margins.
Aids Permitted:	None. *No* calculator.
Advice:	Read problems carefully before jumping in to calculations. Well-drawn sketches / diagrams can be very helpful. If you can't answer a question, you *will* receive part marks for relevant statements, insights, or sketches. Tell me what you know!

Bonus Question

Do **NOT** waste your time here unless you're finished with the exam ...

In class I made some reference to eigendecompositions etc., including that eigenvectors should be orthogonal. Why might this be a good assumption?

Well, given any symmetric matrix A, prove that eigenvectors corresponding to different eigenvalues *must* be orthogonal.

Question 1. (45 Marks)

a) What do we mean by the impulse response of a continuous-time system $T[\,]$?

b) Define, in a sentence or two, what it means for a system to be

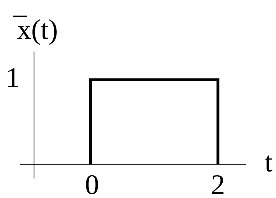
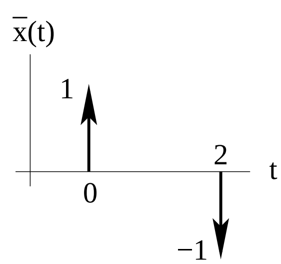
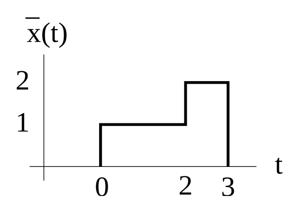
1. Stable
2. Invertible
3. Linear

c) Given a discrete-time impulse response $h(n)$, what are the conditions on $h(n)$ for ...

1. The associated system to have memory?
2. The associated system to be causal?
3. The associated system to be stable?

d) Each of the following three cases describes an LTI system by giving you input $x(t)$ and output $y(t) = T_i[x(t)]$. For each system:

- Give the mathematical form of the new input $\bar{x}(t)$.
- Find the response $\bar{y}(t) = T_i[\bar{x}(t)]$ to the given input $\bar{x}(t)$.
- Draw a neat, labelled sketch of $\bar{y}(t)$.

System	Input $x(t)$	Output $y(t)$	New Input $\bar{x}(t)$
T_1	$\delta(t)$	$e^{-t}u(t)$	
T_2	$\delta(t-1)$	$e^{-\lvert t \rvert}$	
T_3	$u(t)$	$te^{-t}u(t)$	

Question 2. (30 Marks)

We are given an LTI system

$$y(t) = T\left[x(t)\right]$$

with the following impulse response:

$$h(t) = 2\big(u(t) - u(t-3)\big)$$

a) Sketch $h(t)$.

b) Is the system causal? Is it stable? Why / why not?

c) If $x(t) = \frac{1}{2}e^{-|t|}u(t)$, what is $y(t)$?

d) Suppose we connect two such systems in series:

$$z(t) = T\big[T\left[x(t)\right]\big] = T_2\left[x(t)\right]$$

Derive the impulse response $h_2(t)$ of the combined system T_2. Draw a sketch of the impulse response.

Question 3. (30 Marks)

a) Given the continuous-time causal LDE

$$y''(t) - \tfrac{1}{2}y'(t) - \tfrac{1}{2}y(t) = x(t)$$

Derive and sketch the impulse response to this system. Is the system stable?

b) Continuing on from part (a), if we now consider the system

$$y''(t) - \tfrac{1}{2}y'(t) - \tfrac{1}{2}y(t) = x'(t) - \tfrac{1}{2}x(t)$$

Derive and sketch the impulse response to this system.

Draw the Form II realization of this system (using derivatives or integrals, your choice).

c) Given the discrete-time causal LDE

$$y(n) - \tfrac{1}{2}y(n-1) - \tfrac{3}{16}y(n-2) = x(n)$$

Derive the impulse response to the system. Is the system stable?