

Properties of the z-Transform

Property	Signal	z-Transform
	$x[n]$	$X(z)$
	$x_1[n]$	$X_1(z)$
	$x_2[n]$	$X_2(z)$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$
Time shifting	$x[n - n_0]$	$z^{-n_0} X(z)$
Scaling in the z-domain	$e^{j\omega_0 n} x[n]$	$X(e^{-j\omega_0} z)$
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$
	$a^n x[n]$	$X(a^{-1} z)$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ <i>for some integer r</i>	$X(z^k)$
Conjugation	$x^*[n]$	$X^*(z^*)$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$
First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - z^{-1}} X(z)$
Differentiation in the z-domain	$nx[n]$	$-z \frac{dX(z)}{dz}$

Initial Value Theorem

If $x[n] = 0$ for $n < 0$, then

$$x[0] = \lim_{z \rightarrow \infty} X(z)$$