

## Final Examination

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Aids Permitted: *One* 8.5x11 page (both sides), *No* calculator.  
Please turn off cellphones, pagers, ...

\*\*\* Well-drawn sketches / diagrams can be very helpful. \*\*\*

The grade value for each question is indicated in brackets [ ] next to the question number. I *will* give part marks for relevant statements or insights.

Manage your time!!! This is the most common problem which I see among students.

The exam is out of 100  
The exam is 120 minutes long

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[2%] Bonus Question

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Do **NOT** waste your time here unless you are happy with your answers to the rest of the exam!!!

To make it easier for me to find, an answer to the Bonus question must appear on the very back page of an exam booklet.

I take a bag of frozen peas out of my freezer (-25C) and put them into the fridge (+4C) to thaw. By putting them into the fridge I also keep the fridge cool and save energy. To my disappointment, a short while later I notice the fridge running, even though I notice that the peas are still frozen.

- Why is the fridge running?
- How is this phenomenon related to global energy issues?

1. Explain (one paragraph) the role and importance of data assimilation and inverse problems in the context of global climate models.
2. What is meant by a *stiff* ODE or PDE? What problems does stiffness lead to? What are one or two methods for dealing with stiffness?
3. Why are there *no* truly linear systems in the natural world?

4. For the system

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

- a) Is the system linear? Why / why not?
- b) What class of dynamic behaviour does this system possess?
- c) Draw the phase portrait.

5. For each of the following dynamic matrices ...

$$\begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 5 \\ 1 & -2 \end{bmatrix} \quad \begin{bmatrix} -3 & 2 \\ 1 & -1 \end{bmatrix}$$

- a) Identify the behaviour type.
- b) If the dynamic was the result of linearizing a nonlinear system, what could you conclude about the nonlinear system at that point?

6. Given the following nonlinear system

$$\begin{aligned} \dot{x} &= x^3 - x \\ \dot{y} &= \frac{1}{1 + e^{-y}} - \frac{1}{2} \end{aligned}$$

- a) Identify the fixed points of the system.
- b) At each fixed point, find the linearized system and the type of dynamic behaviour.
- c) Draw the phase portrait.

7. Suppose we consider UW as a system, with a system envelope drawn around the physical campus. Draw a system diagram identifying the main energy, resource, and other physical flows (everything *except* the “flow” of people).
  
8. Looking at agricultural nutrients from a systems level perspective, identify and discuss (in a few sentences) two policy issues related to one or more of nitrogen / potassium / phosphorus.
  
9. ERS1/2 were a “tandem pair” — two identical radar satellites flown at a precise orbital offset. Explain . . .
  - a) The imaging principle behind such a pair — what does such a pair allow us to do, and how does it work?
  - b) The applied principle behind such a pair — what are the earth systems phenomena that can be observed?
  
10. Two well-known altimeters:
  - ERS-1: A 35-day exact repeat cycle
  - Topex-Poseidon: A 10-day exact repeat cycleAnswer the following:
  - a) Define what is meant by an “exact repeat cycle”
  - b) What are the pros and cons of 10 versus 35 days?
  - c) What repeat cycle periods should be avoided and why?
  
11. We can study self-organized criticality by computer simulation on a grid.
  - a) Why is SOC so commonly studied via simulation, instead of analytically (mathematically)?
  - b) Explain why it is essential that the grid be large.