SD 675 Pattern Recognition

Assignment 1
(Assignments are to be done individually. Do not write a formal report.)

Purpose

This lab investigates eigendecompositions and orthonormal transformations.

Assignment

We have studied orthogonal whitening transformations (GED), however given two class covariances where $S_1 \neq S_2$, normally the transformation which whitens S_1 (i.e., transforms $S_1 \Rightarrow I$) does *not* diagonalize or whiten S_2. Interestingly it *is* possible, however, to diagonalize both classes with a single transformation.

Suppose we have the eigendecomposition $S_1 = V_1 \Lambda_1 V_1^T$.

a) Find T_1, the orthonormal whitening transformation for class 1.

   Show that $T_1 S_1 T_1^T = I$.

b) Let $K = T_1 S_2 T_1^T$, the application of T_1 to class 2.

   Suppose we are given the eigendecomposition $K = V_k \Lambda_k V_k^T$, what is the transformation T_k to diagonalize (but not whiten) K?

   Verify that

$$T_k T_1 S_1 T_1^T T_k^T = I \qquad\qquad T_k T_1 S_2 T_1^T T_k^T = \text{diagonal}$$

   That is, we have fully whitened class 1 and diagonalized class 2.

c) We could have done all of the above in one step. Let

$$A = T_k T_1$$

   be the net transformation from (a) and (b), above. Show that A satisfies the generalized eigenequation

$$S_1^{-1} S_2 A^T = A^T \Lambda$$

   for some diagonal matrix Λ. This shows that it is possible to find the net transformation A directly from a single eigendecomposition.

d) Would it be possible to *whiten* (and not just diagonalize) *both* classes simultaneously?
   Why / why not?

   Draw a neatly labelled sketch to illustrate the actions of T_1, T_k from (a) and (b).

   Your sketch should clearly illustrate why simultaneous whitening is / is not possible.