SD 675 Pattern Recognition

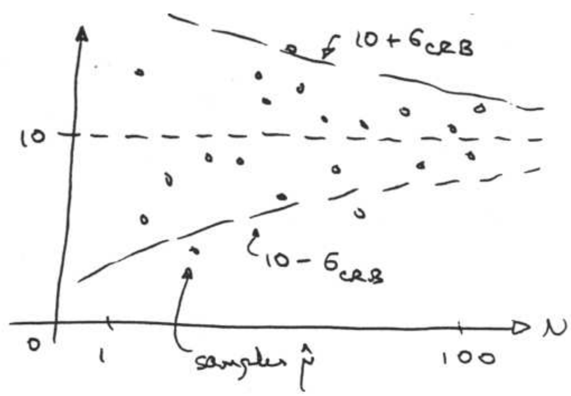
Winter 2009

Parameter Estimation and CRB.

Suppose we have N independent samples $x_1, \ldots, x_N$ of a Gaussian random variable $x \sim \mathcal{N}(\mu, \sigma^2)$. We want to estimate the mean, μ.

1. Derive the ML estimator $\hat{\mu}_{ML}(x_1, \ldots, x_N)$.
2. Derive the estimation error variance σ_e^2 as a function of N and σ^2.
3. Derive the CRB σ_{CRB}^2 for the error variance in estimating μ. That is, find the bound on σ_e^2 as a function of N and σ^2.
4. Now say that $\mu = 10, \sigma^2 = 10$. For $N = 1, 2, \ldots, 100$ simulate the above three steps and plot the results.

   That is, for each value of N, generate random values x_i, calculate $\hat{\mu}$, and plot $\mu, \hat{\mu}, \mu \pm \sigma_{CRB}$ as sketched.

5. Next, if we didn't actually know the value of σ^2, then we also don't actually know the value of σ_e and σ_{CRB}. So we're motivated to estimate σ_e from the data. So we can generate N random values to find $\hat{\mu}$; then we can do the preceding M times, yielding $\hat{\mu}_1, \ldots, \hat{\mu}_M$, from which we can calculate a sample variance.

   Of course, you're not actually getting σ_e^2 exactly; the sample variance $\hat{\sigma}_e^2$ is an *estimate* of σ_e^2.

   So now I want you to derive the CRB for the error variance in estimating σ_e.

6. Again we want to simulate this for $N = 1, 2, \ldots, 100$, for two values of M: $M = 5,\ M = 50$. For each value of M, produce a plot superimposing the true σ_e, the estimated $\hat{\sigma}_e$, and $\sigma_e \pm \sigma_{e-CRB}$.

Give a brief interpretation / discussion of the results.