

### Example 7.6: Limit Cycles in Economics

The *Business Cycle* is one of the most fundamental concepts in macro-economics, describing the large-scale swings observed in economic activity.

Certainly there are repeated cycles in history where people become increasingly confident, undertaking more and more leveraged investments (investing money you don't have, by borrowing); at some point the borrowed money *does* need to be paid back, thus a few people sell, and then more, leading to a cascading effect (catastrophe) whereby many people lose money and have a greatly reduced tolerance for risk. Some of the most famous and extreme events include the 17<sup>th</sup> century Tulip Mania, the 18<sup>th</sup> century South-Sea Bubble, the 1920's stock market, and the turn-of-the-century Dot-Com Bubble.

We do not need to limit our attention to such extreme events to observe cycling behaviour. The *Goodwin Model* is one example of a quantitative economic dynamic model relating unemployment and wages. In its simplest form, the model has two coupled state variables

Employment rate  $v$       Wages  $u$

where  $u$  represents the fraction of output being returned to workers. The Goodwin Model then takes the form

$$\dot{v} = v(\alpha(1 - u) - \beta) \tag{7.18}$$

$$\dot{u} = u(\rho v - \gamma) \tag{7.19}$$

where, in a nutshell, (7.18) argues that higher wages  $u$  leave less room  $(1 - u)$  for capital investment, leading to reduced employment. Whereas (7.19) asserts that high employment (low unemployment) leads to an upwards pressure on wages.

However (7.18), (7.19) are, yet again, in the **predator-prey** class of models, consisting of a centre fixed point surrounded by cycling behaviour. Thus to the extent that the Goodwin model is correct, it suggests that governments' desire for high wages and high employment (i.e., not at the fixed point) faces a never-ending battle, since away from the fixed point the system is inclined to cycle, and can be held at that point only with constant **control**.

To be sure, there are many other models which have been proposed or claimed to describe social and economic cycles ...

Name of Cycle	Associated Context	Cycle Period in Years
Kitchin cycle	Inventory	3 to 5
Juglar cycle	Fixed investment	7 to 11
Kuznets swing	Infrastructure investment	15 to 25
Kondratiev wave	Technologically driven	45 to 60
Elliot wave	Investment psychology	≪ 1 to 100

A number of these models or claimed behaviours are somewhat controversial, and are really more of an observation of past cycling behaviour than a verifiable model or theory. Nevertheless, the presence of economic cycling is very real, with significant implications in terms of employment, social unrest, policy, and mal-investment.

Further Reading: Predator-prey models are simulated in [Problem 7.16](#).

Also see [Economic Bubble](#), [Goodwin Model](#), [Business Cycle](#), [Elliott Wave](#), [Kondratiev Wave](#) in *Wikipedia*.